5. Set theory

Definition: A **set** is a collection of objects. The objects are called elements or points of the set.

A set is usually denoted by a capital letter , e.g. A,B,C,…

If w is a point or element belonging to the set A , we shall write $w \in A$, if w is not an element of A , we shall write $w \notin A$.

Definition: The set A is said to be a **subset** of B , if every element of A is also an element of B. this is denoted by $\boldsymbol{A \subset B}$.

Example: Let $A = \{1,2,3,4\}$ *and* $B = \{x: x \text{ is a positive integer and } x < 8\}$, *then*

$B = \{1,2,3,4,5,6,7\}$,

$\therefore A \subset B$

Definition: A set is said to be an **empty** set or the **null** set, if it has no elements, and is denoted by $\emptyset$. For any set, we have $\emptyset \subset A \subset S$.

Definition: Two sets A and B are said to be **equal** set , if $A \subset B$ and $B \subset A$, and we write A=B

Equal Set Example: If P = {1, 3, 9, 5, −7} and Q = {5, −7, 3, 1, 9,}, then P = Q.

Definition: The **complement** of the set A with respect to the space S, denoted by $\acute{A}$, A^c , is the set of all elements that are in S but not in A, i.e.

$$\boldsymbol{\acute{A} = \{x: x \in S \text{ and } x \notin A\}}$$

Example: If A = { 1, 2, 3, 4} and U = { 1, 2, 3, 4, 5, 6, 7, 8} then find A complement (A’).
Solution : A = { 1, 2, 3, 4} and Universal set = U = { 1, 2, 3, 4, 5, 6, 7, 8}
Complement of set A contains the elements present in universal set but not in set A. Elements are 5, 6, 7, 8.
∴ A complement = A’ = { 5, 6, 7, 8}.

Example: If A = { x | x is a multiple of 3, x ∉ N }. Find A’.
Solution : As a convention, x ∉ N in the bracket indicates N is the universal set.
N = U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,11, … }
A = { x | x is a multiple of 3, x ∉ N }

A = { 3, 6, 9, 12, 15, … }
So, A’ = { 1, 2, 4, 5, 7, 8, 10,11, … }

Definition: Let A and B be any two subjects of S. the set of all points in A that are not in B will be denoted by $A|B$ or A-B and is defined as set **difference**

$$\boldsymbol{A|B = \{x: x \in A \text{ and } x \notin B\}}$$

Not that $\acute{A} = S|A$

Example: Let S be the set of all natural numbers , i.e. S=N={1,2,3, ... }

Let $A = \{x: x\ is\ an\ even\ number, x \in N\}, and\ B = \{y: y\ is\ a\ multiple\ of\ \ 3 , y \in N\}$

Then find $\acute{A}$, $\acute{B}$, $A|B$, $B|A$.

Solution :

$A = \{2,4,6,8,10,12,14, \dots\}$

$B = \{3,6,9,12,15,18, \dots\}$

$\acute{A} = \{1,3,5,7,9, \dots\}$

$\acute{B} = \{1,2,4,5,7,8,10,11, \dots\}$

$A|B = \{2,4,8,10,14,16,20, \dots\}$

$B|A = \{3,9,15,21,27, \dots\}$

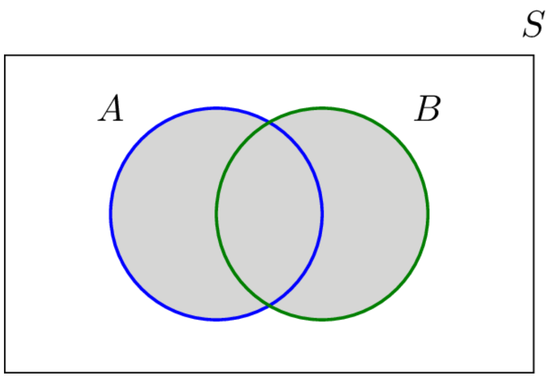
Definition: Let A and B be any two subsets of the space S. The **union** of A and B, denoted by $A \cup B$, is the set of all elements belonging either to A or to B, including those which belong to both, i.e.

$A \cup B = \{x: either\ x \in A\ or\ x \in B\ or\ in\ both\}$

$x \in A \cup B\ if\ and\ only\ if\ x \in A\ or\ x \in B$

$x \notin A \cup B\ if\ and\ only\ if\ x \notin A\ or\ x \notin B$

Example: The union of sets A and B is shown by the shaded area in the Venn diagram.



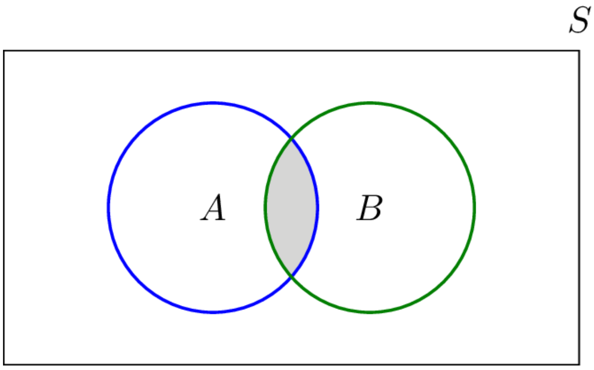
Definition: The **intersection** of A and B, denoted by A$\cap B$, is the set of all elements that are in both A and B, i.e.

$A \cap B = \{x: x \in A\ and\ x \in B\ \}$

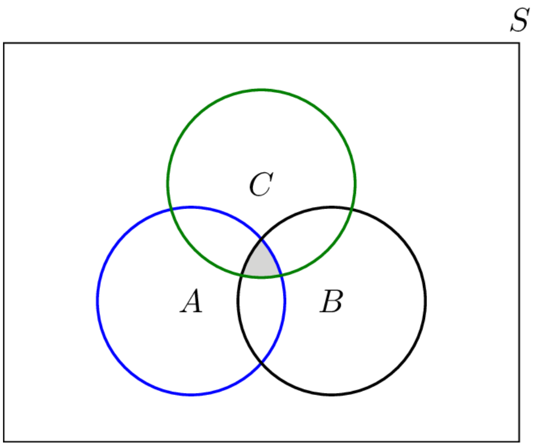
$x \in A \cap B\ if\ and\ only\ if\ x \in A\ and\ x \in B$

$x \notin A \cap B\ if\ and\ only\ if\ x \notin A\ and\ x \notin B$

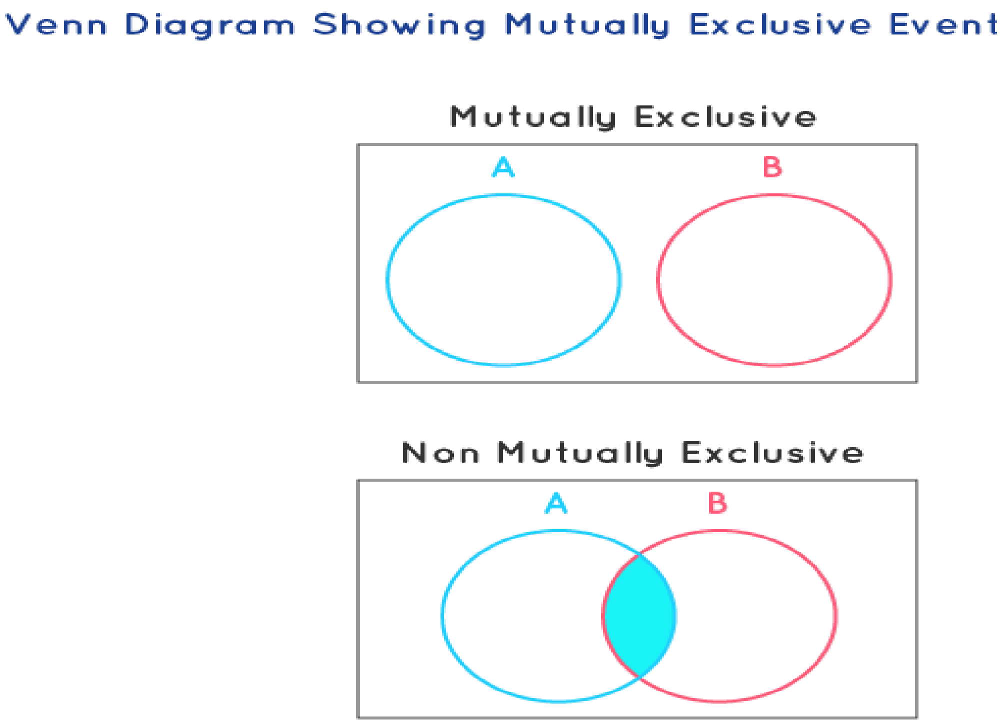
Example: The intersection of sets A and B is shown by the shaded area using a Venn diagram.

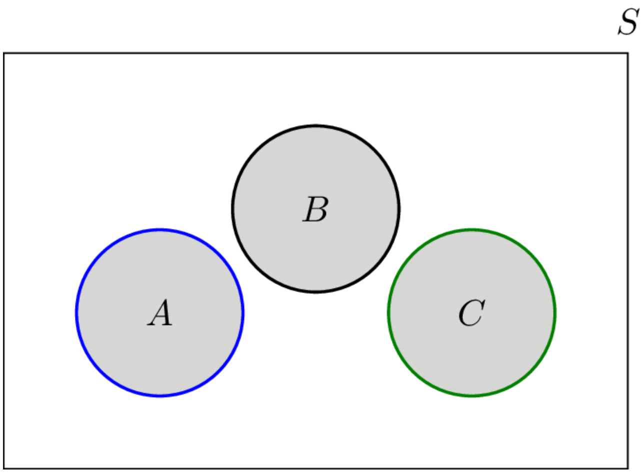


The intersection of three sets A, B, C.



Definition: Two sets A and B are said to be **disjoint or mutually exclusive**, if they have no common elements, i.e. $A \cap B = \emptyset$





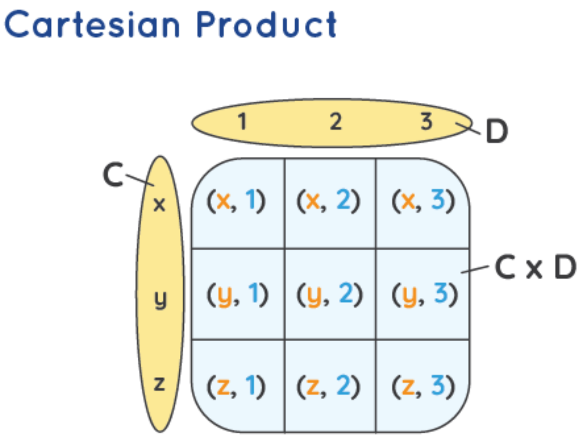
Sets A,B, and C are disjoint

Definition: Let A and B be two sets. The **product** set of A and B, denoted by $A \times B$, is the set of all ordered pairs (x, y) where $x \in A \; and \; y \in B$, i.e.

$A \times B = \{(x, y): x \in A \; and \; y \in B\}$

How to Find Cartesian Product?

Consider two non-empty sets C = {x, y, z} and D = {1, 2, 3} as shown in the below image:



C x D = {(x,1), (x,2), (x,3), (y,1), (y,2), (y,3),(z,1), (z,2), (z,3)}.

Example:Let us find the Cartesian product of the two sets C and D, where C = {11,12,13} and D = {7, 8}. After following the steps mentioned above:

- The resultant product
  C x D = {(11,7), (11,8),(12,7),(12,8),(13,7),(13,8)}.
- Similarly, we can find the Cartesian product of D and C as
  D × C = { (7,11),(7,12),(7,13),(8,11),(8,12),(8,13)}.
- The Cartesian products C × D and D × C do not contain exactly the same ordered pairs. Hence, in general, C × D ≠ D × C.